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PROBABILISTIC STRUCTURAL INTEGRITY

N. A. Zentuti[1], J. D. Booker[1+], J. Hoole[1], R. A. W. Bradford & D. Knowles[1]

Solid Mechanics Research Group, Faculty of Engineering, University of Bristol[1]
([+]Corresponding Author Email: j.d.booker@bristol.ac.uk)

Abstract

This paper highlights the wide range of applications for probabilistic analyses in structural integrity and design. Firstly, some background is given to introduce the basic principles which underpin a probabilistic approach and some aspects of its historical development. A discussion of the main benefits and attributes of probabilistic analyses follows, and a direct comparison with conventional deterministic design is made. Thereafter, five main modes of application are presented, and their individual objectives discussed. Through examining various case studies, the different utilities of a probabilistic approach are highlighted. Finally, some important considerations are discussed in terms of adopting probabilistic approaches for future implementation.

Keywords: *probabilistic approach, structural integrity, case studies, implementation issues.*

1. Introduction

Probabilistic approaches for design and structural integrity analysis have been with us for over 50 years now and have been found to facilitate a more realistic understanding of performance of components, products and systems through the incorporation of uncertainties in input parameters. Historically, these uncertainties have been catered for using large factors of safety (sometimes termed factors of ignorance) in a deterministic approach. A deterministic approach may result in inconsistent designs (over- or under-design) owing to limitations in the information about design, manufacturing, material and service related parameters (1). Conversely, in a probabilistic approach, methods are used to investigate the combination and interaction of these input parameters, having characterised distributions, to estimate the probability of failure, for optimisation and to perform sensitivity studies. Probabilistic approaches are said to be the only established methodology of propagating uncertainty through engineering models (2), yet they have not been taken up routinely by industry, and a deterministic culture still dominates many engineering domains to date.

A brief review of the use of factors of safety in practice is an important step in understanding why probabilistic approaches were developed. The explicit use of safety factors in calculations started circa 1850s and continued unchallenged for about 100 years (3). Lower limits of strength and upper limits of loading stress were typically applied in anticipation of uncertainty, but with safety factors applied on top of these judgments to account for other uncertainties associated with unknown factors, in particular, for modelling inaccuracies. Safety factors were also developed for different areas of application as domain specific expertise and experience guided (4). As engineers learned more about the nature of variability in engineering parameters in general, but were unable to quantify these satisfactorily, naturally factors of safety increased with time (5). The factor of safety had little scientific background, having an underlying empirical and subjective nature. No one can dispute that at the time that, say, stress analysis was in its infancy, this was the best knowledge available, but there was also the fact that there were a sufficient number of failures still happening to conclude that deterministic approach does not always ensure intrinsic reliability (6). A great disenchantment with factors of safety grew over the years (7).

As early as the 1920s, however, it was suggested that design performance should be based on means and variances of the random variables involved (8); a totally different mind-set at the time. In the 1940s, a statistical basis to the factor of safety was proposed (9)(10). In the 1950s, engineers began to think differently about design, say, with the introduction of a '*True margin of safety*', demonstrating that engineering problems are multi-factored, and variability based (11). With the increasing use of statistics in engineering around this time, and the advent of mass production, these ideas were the real enablers for probabilistic approaches, and in the 1960s, the theoretical developments governing probabilistic design, its paradigms and methods were becoming established (4)(12)(13)(14). Many practical methods have since been developed to cope with the full range of engineering problem types, model formulations and data characteristics (15).

In this paper, the basic underlying principles of the probabilistic approach are presented, and a comparison made to the deterministic approach, illuminating the former's benefits. A classification of the types of application, or utilities, of probabilistic methods is presented, followed by a number of design and structural integrity case studies which are used as exemplars for the specific application objectives: reliability prediction, performance assessment, optimisation, sensitivity analysis and uncertainty characterisation. The paper closes with a discussion of the implementation issues surrounding probabilistic approach, before conclusions are presented.

2. The Basic Principles

Virtually all engineering parameters such as dimensions, material properties and service loads exhibit some statistical variability that influence the adequacy of the problem, and therefore should be treated as random variables. The main engineering random variables that should be adequately described when using a probabilistic approach are shown in Figure 1. The variables conveniently divide into two types: *design dependent*, which the designer has the greatest control over, and *service dependent*, which the designer has 'limited' control over.

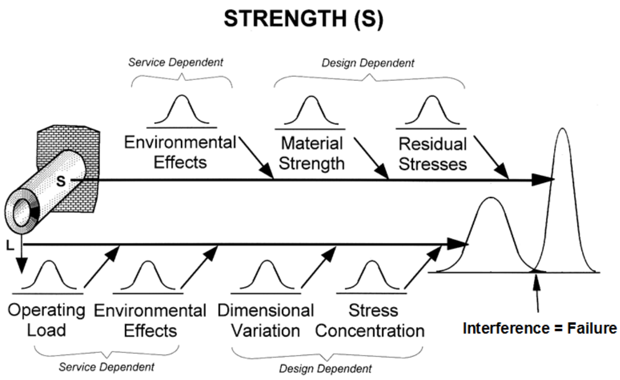

Figure 1: Key Variables in a Probabilistic Design Approach (5)

Typically, the most important design dependent variables are material strength and dimensional variability. Material strength can be statistically modelled from sample data for the property required, however, difficulties exist in the collation of information about the properties of interest. Dimensional variability and its effects on the stress acting on a component can be great, but information is typically lacking about its statistical nature, and its impact on geometric stress concentration values are rarely assessed. Important service dependent variables are related to the loading of the component and stresses resulting from environmental effects. These are generally difficult to determine at the early stages of a project because of the cost of performing experimental data collection, the nature of overloading and abuse in service, and the lack of data about service loads. Also, the effect that service conditions have on the material properties is important, the most important considerations arising from extremes in temperature, as there is a tendency towards brittle fracture at low temperatures, creep rupture at high temperatures.

Several researchers and organisations over the last few years have accumulated statistical data for important engineering properties. Table 1 shows the scale of the variability of these parameters in terms of the coefficient of variation, C_v (the standard deviation divided by the sample mean) (5). If variability is said to be a major source of unreliability in an engineered system or product (16), then observing the data in Table 1 suggests that the quality control of the manufacturing processes and material properties might not be as important as controlling the service environment and loads in achieving high reliability (6). The random nature of the engineering parameters and the scale of the variation in each is therefore well known. Engineers are familiar with the typical appearance of sets of strength data from tensile tests in which most of the data values congregate around the mid-range with decreasingly fewer values in the upper and lower tails on either side of the mean. For mathematical tractability, the experimental data can be modelled with a Probability Density Function (PDF) or *continuous distribution* that will adequately describe the pattern of the data using just a single equation and its related parameters e.g. Normal, Weibull, Lognormal and Extreme Value types, to name just a few.

Table 1 Typical coefficient of variation, C_v for a variety of engineering parameters (5)

Loads	**Geometry Parameters (produced by manufacturing processes)**
Aerodynamic loads in aircraft = 0.012-0.04	Grinding = 0.00015
Spring force = 0.02	Turning/boring = 0.0004
Bolt preload using powered screwdrivers = 0.03	Powder metal sintering = 0.0006
Aircraft thrust loads = 0.05	Drilling = 0.001
Thermal loads = 0.08	Milling = 0.003
Powered wrench torque = 0.09	Hot rolling = 0.008
Dead load = 0.1	Closed die forging = 0.009
Hand wrench torque = 0.1	
Vibration loads = 0.2	**Other Engineering Parameters**
Live load = 0.25	Coefficient of linear expansion for metals = 0.01
Snow load = 0.26	Stress concentration factor for machined notched bar = 0.03
Human arm strength = 0.3-0.4	Surface tensile residual stress for turning/boring = 0.1
Wind loads = 0.37	Nut factor for cadmium plated bolt = 0.12
Acoustic loads in aircraft = 0.4	Surface roughness for turning/boring = 0.2
Transient loads = 0.5	Coefficient of friction for steel on steel shrink-fit = 0.22
Material Properties	
Steels	***Other Materials***
Ultimate Tensile Strength = 0.05	Ultimate Tensile Strength for cast iron = 0.09
Yield Strength = 0.05-0.2	Ultimate Tensile Strength for wrought iron = 0.04
Endurance Strength = 0.08	Rupture strength for carbon fibre composites = 0.17
Brinnel Hardness = 0.05	Modulus of Elasticity for cast iron = 0.04
Modulus of Elasticity = 0.01-0.03	Modulus of Elasticity for titanium = 0.09
Modulus of Rigidity = 0.02-0.04	Modulus of Elasticity for aluminium = 0.03
Fracture Toughness = 0.05-0.1	Shear and compression strength for honeycomb = 0.1
Poisson's Ratio = 0.025	Tensile strength for honeycomb = 0.16
	Ultimate tensile/yield strength for non-ferrous metals = 0.05

Composing these statistically characterised parameters in the performance model which reflects the objective of the problem, we must then use certain probabilistic methods to essentially simulate large numbers of random events taking place. This argument is extended when considering the probability of failure of a component, which is based on the joint probability of interference of the inherent material strength distribution (S), and loading stress distribution, (L), where both are random variables (see Figure 1). When a random stress exceeds a random strength on any simulation event, failure is recorded, and this continues for all events until a probability of failure can be predicted accurately enough. In essence, the interference between the actual stress and strength distributions dictates the performance of the product in service and this is the basis of the probabilistic approach. Accurate representations of stress and strength as distributions, therefore, enables a meaningful failure prediction to be generated.

3. Why a Probabilistic Approach?

As outlined from the above arguments, a probabilistic approach incorporates the uncertainties with typical design inputs fully using any experimental data available, and thus provides the required realism when modelling a problem (1). It also provides quantitative measures of performance, helping engineers draw conclusions from complex analyses with a high degree of confidence. Increasing demands for higher performance and efficiency, resulting often in operation near limit conditions, has placed increasing emphasis on precision and realism in this way (17)(18). Increased use of analytical and simulation techniques in engineering is facilitating a move to fully simulation-based design, which for required confidence, must also be conducted probabilistically. This will eventually lead to cost savings on prototype validation prior to the production and installation of engineered products and systems. But probabilistic approaches are also useful where test to failure is not a practical proposition, where weight minimisation and/or material cost reduction is important (i.e. not overdesigned), or where a safety case has to be made e.g. in a component structural integrity assessment for operational plant. The only alternative is to resort to a deterministic approach using Factors of Safety, which can lead to either an unconservative design with unacceptable high failure rates, or a very conservative design that provides the required performance with unnecessarily high costs (19). Either way, determinism is conservatism, which is ineffective for the modern world (20). Table 2 summarises some of the important characteristics of probabilistic approach compared to a deterministic one.

4. Case Studies

A number of important applications of probabilistic methods exist in reliability prediction, as discussed earlier, specifically where it would be useful to explore the level of random failure, resulting from the interaction of the distributions of loading stress and material strength. Other benefits from building a probabilistic model also present themselves however, for example, the purpose of a sensitivity analysis is to provide a measure of the contribution of each of the input design and service dependent parameters to the problem. Only a few parameters have a significant impact on output variability generally (i.e. Pareto's Rule analogy) and can lead to a better characterisation of the variability in the most influential parameters, with a focus on quality control and increased data collection and characterisation of the properties of interest and influence. This can also speed up a probabilistic assessment by negating trivial parameters in the problem formulation and computation. Sensitivity analysis can be snapshot (time independent) or time dependent, if the operating or environmental conditions change with time. Other utilities are to validate the output of a model or several models against experimental results with confidence measures and to provide optimisation of the parameters involved in the problem given some target e.g. probability of failure. Table 3 shows the main utilities of a probabilistic approach.

Table 2 Deterministic versus Probabilistic Approach

	Deterministic	Probabilistic
Inputs	Single values e.g. lower bounds	Distributions, histograms, ranges or single values
Outputs	Pass/fail result	Probability of failure (single value)
Correlations	Based on judgment	Measured from data
Sensitivity study	Local	Local and global
Run time	Single runs	Typically $> 10^5$ runs

Table 3 Main Utilities of a Probabilistic Approach

Application	Objective(s)
Reliability/risk assessment	Evaluate the probability of failure (or reliability).
Performance assessment	Given a design what range performances can I expect? Provides measures for improvement.
Optimisation	Reduce redundancies through more economic design. Optimise a design given multiple competing requirements.
Sensitivity analysis	Which inputs contribute the most to the output uncertainty.
Characterisation of uncertainties	Increase confidence in processes and outputs.

Figure 2 overviews the probabilistic analysis of a forged steel foot pedal under quasi-static loading (1,000 load applications), with the objective of finding the section depth for intrinsic reliability (21). The outcome suggests this is achieved at 22 mm section depth, compared to 33 mm from a deterministic approach using a Factor of Safety = 3, typical for this type of problem. The potential for overdesign and excessive use of material and increased cost is evident using the deterministic approach. Within the topic of fatigue design, a number of parameters exhibit variability, including the scatter present within S-N data, uncertainty within the loading spectrum and dimensional variability (22) (see Figure 3). This leads to variability in the accumulated fatigue damage computed by Miner's Rule which can be simulated using a Monte Carlo Simulation. As the failure criterion for Miner's Rule also exhibits variability (23), a stress-strength interference approach can be used to compute the probability of failure of the component, to support design decisions. Figure 4 presents the probabilistic assessment of three different theories for shrink-fit holding torque compared to the experimental distribution composed from 27 samples; the preferred theory being number 3, which is 'closer' to the experimental, but is statistically verified as being the same distribution at a certain confidence level (26).

Similar to the previous case in its objective, Figure 5 shows the experimental creep damage results for a specimen under uniaxial loading compared with two competing probabilistic models for creep relaxation. A greater degree of confidence is related to the use of Model 2 compared to Model 1 (27). The solenoid design shown in Figure 6 has two failure modes: failure potential at the weakest section by stress rupture due to the assembly torque, and the second is that the pre-load on the solenoid thread section is sufficient to avoid loosening in service. It is necessary to determine the mean assembly torque, M, to satisfy these two competing failure modes using a probabilistic design approach, where the target torque to be applied is an optimum at the highest reliability achievable. Figure 7 shows the Sensitivity Analysis results from the relative contribution of six input parameters, which are all described as random variables, towards the output of a creep-fatigue damage assessment, using four different Sensitivity Analysis methods (27). Using four different methods (local, global) allows corroboration of the results, although, the main interest is gauging which parameters require further data collection, quality control etc. Figure 8 shows a time dependent Sensitivity Analysis and the shift in the relative importance of two input parameters owning to the time dependent nature of creep. Figure 9 provides the probabilistic creep-fatigue damage results as compared with the outcome of a conventional deterministic calculation of the same type, showing the relative conservatism of the latter. Figure 10 shows the effect of correlations between input creep properties on probabilistic creep-fatigue damage results.

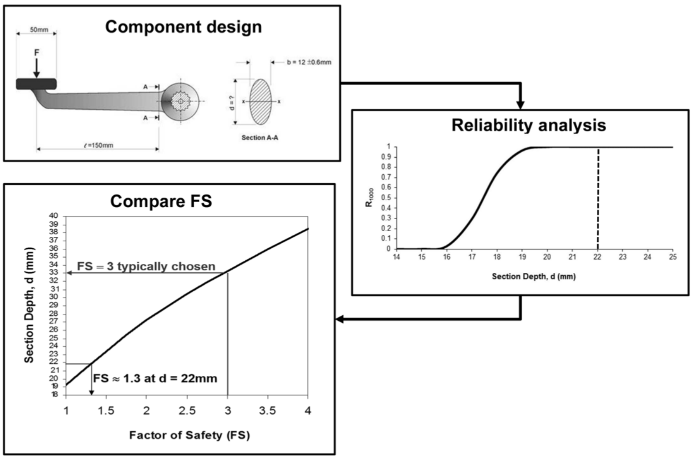

Figure 2: Reliability/Risk Analysis – Component Quasi-static Design (21)

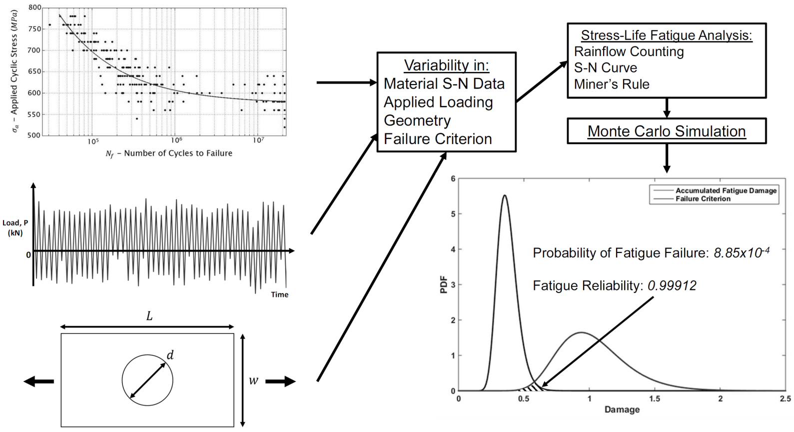

Figure 3: Reliability/Risk Analysis – Component Fatigue Design (24)(25)

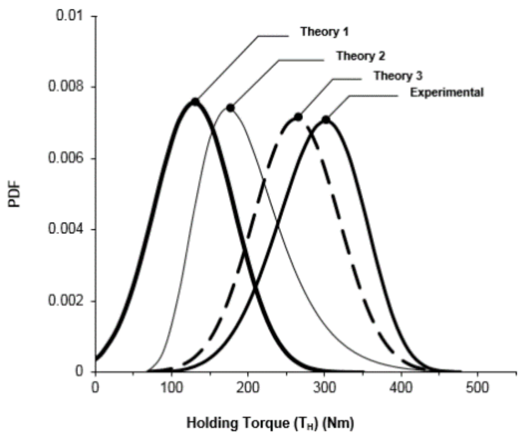

Figure 4 Performance Assessment – Shrink-fit Design (26)

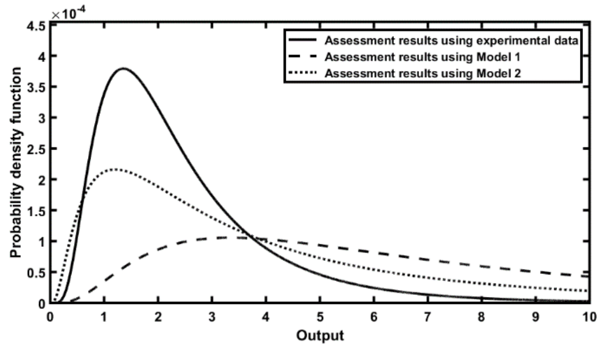

Figure 5 Performance Assessment – Creep Damage (27)

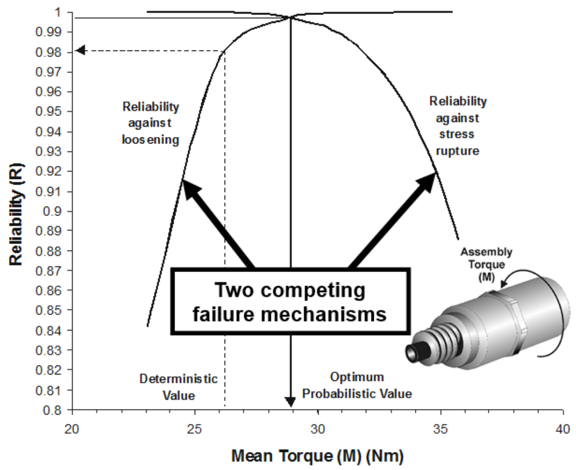

Figure 6 Optimisation – Competing Failure Modes (5)

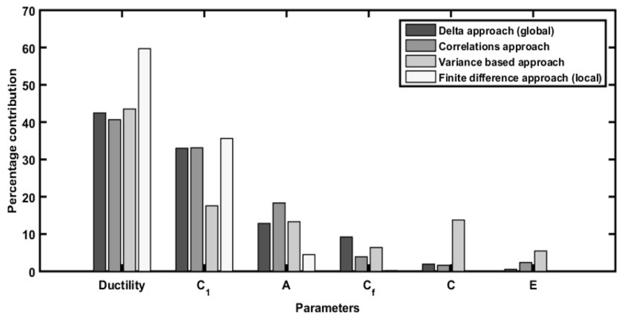

Figure 7: Sensitivity Analysis - Uniaxial Creep-Fatigue Assessment (27)

5. Important Considerations

It must be remembered that anything which is deterministic can be modelled probabilistically as well; though more time, effort and cost is involved to do so generally. Much more information is needed for a probabilistic approach than for a deterministic one though, self-evidently. Moreover, it has been argued that probabilistic approach can be used only when all the needed statistical data is available, and it would be misguided to use it otherwise (28). Certain additional demands are made of the designer or engineer in order to conduct a probabilistic approach over a deterministic one, and these can really be summarised into just two major issues: have some statistics understanding, and the ability to generate efficient computational algorithms.

The additional time required over conventional deterministic approaches can be traded off against the accuracy of the probabilistic model required and the stage in the development cycle you are in. The robustness of the chosen probabilistic methods towards the type of system or problem to be modelled is also an important selection decision i.e. completeness and statistical description of data used, engineering model type (analytical, numerical, empirical, non-linear) and objective to be achieved e.g. optimisation, probability of failure prediction etc (15). For example, the evaluation of many engineering systems requires the use of numerical methods e.g. finite element analysis. Therefore, numerically efficient algorithms are required for probabilistic approaches when applied in order that time and costs are reduced (29).

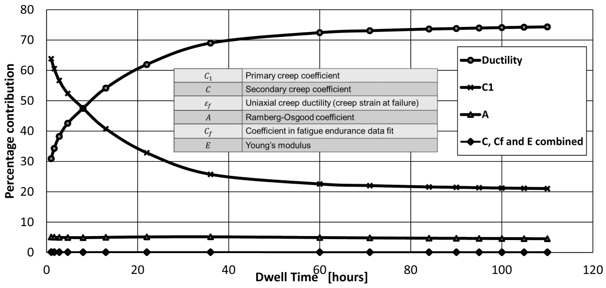

Figure 8: Time Dependent Sensitivity Analysis - Uniaxial Creep-fatigue Assessment

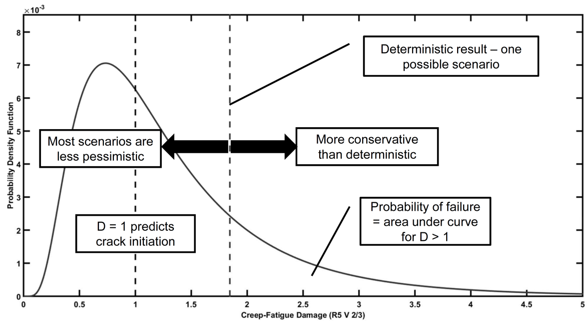

Figure 9: Uncertainty Characterisation - Creep-fatigue Damage

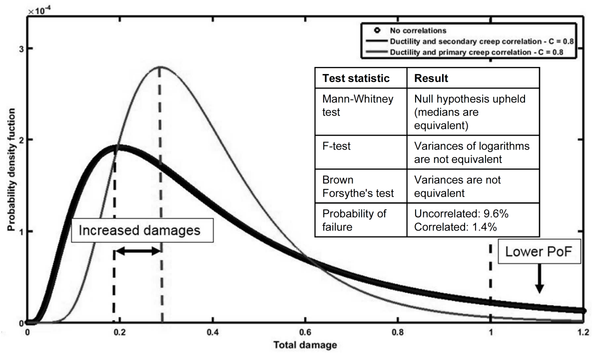

Figure 10: Effect of Correlations on Creep-fatigue Damage

6. Conclusions

The measures of performance determined using a probabilistic approach give the designers more confidence in their assessments, analyses and predications by providing better understanding of the variables involved and quantitative estimates for failure probability. Although associated methods have been adopted in some companies and have been successfully applied to specific cases, a cultural and educational step-change is necessary for probabilistic approaches to be integrated in mainstream engineering activities leading to a probabilistic 'mind-set'. A lack of awareness of their existence or usefulness, computational intensity, requirement for statistical knowledge, and difficulty and cost of data collection, are possibly a few of the obstacles that may have limited their use to date. Industry also often finds it difficult to justify the resource and training commitments needed to support these activities, and a key problem has been in consolidating the knowledge for advancing their utility. Probabilistic design is seen as an opportunity for companies to enhance their competitive advantage through optimisation, reliability and performance target qualification. It provides a transparent means of explaining to a business more about the safety aspects of engineering design decisions with a degree of clarity not provided by the 'factor of safety' approach. Although significant advances have been made in probabilistic methods in terms of efficiency and accuracy, their integration in industrial product development is unsatisfactory. The focus now should now be on the routine use of probabilistic approaches by companies across all sectors. It is hoped that new and existing practitioners alike gain will gain some confidence to progress with probabilistic applications from the examples presented which span a range of different structural integrity and design analysis problems.

Acknowledgements and Dedication

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References

1. Bury, K.V. (1975) Statistical Models in Applied Science. Wiley, New York.
2. Wojtkiewicz, S. F., Eldred, M. S., Field, J. R., Urbina, A., & Red-Horse, J. R. (2001) A toolkit for uncertainty quantification in large computational engineering models. Proc. 42nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials (SDM) Conference, Seattle, Washington, 16–19 April. Paper No. AIAA-2001–1455.
3. Clausen, J., Hansson, S. O. & Nilsson, F. (2006) Generalizing the Safety Factor Approach. Reliability Engineering and System Safety, **91**, 964-973.
4. Madsen, H. O., Krenk, S. & Lind, N. C. (2006) Methods of Structural Safety. Dover, New York.
5. Booker, J. D., Raines, M. & Swift, K. G. (2001) Designing Capable and Reliable Products. Butterworth-Heinemann, Oxford.
6. Carter, A. D. S. (1986) Mechanical Reliability. 2nd Edition, Macmillan, London.
7. Dieter, G. E. (1986) Engineering Design: a materials and processing approach. 1st Metric Edition, McGraw-Hill, New York.
8. Mayer, M. (1926) Die Sicherheit der Bauwerke. Springer Verlag, Berlin.
9. Pugsley, A. G. (1944) A Philosophy of Strength Factors. Aircraft Engineering & Aerospace Technology, **16**(1), 18-19.
10. Freudenthal, A. M. (1947) The Safety of Structures. Transactions of the American Society of Civil Engineers, **112**(1), 125-159.
11. Nixon, F. (1958) Quality in Engineering Manufacture. Proc. of the Conference on Technology of Engineering Manufacture, IMechE, London, **14**, 561-569.
12. Saleh, J. H. & Marais, K. (2006) Highlights from the Early (and pre-) History of Reliability Engineering. Reliability Engineering and System Safety, **91**, 249-256.
13. Lipson, C., Sheth, N., Disney & R. L. (1967) Reliability Prediction - Mechanical Stess/Strength Interference Models (Ferrous), RADC - TR - 66 - 710, March (AD/813574).
14. Haugen, E. B. (1968) Probabilistic Approaches to Design. Wiley, New York.
15. Goh, Y. M., McMahon, C. A. & Booker, J. D. (2009) Improved Utility and Application of Probabilistic Methods for Reliable Mechanical Design. Proc. Instn. Mech. Engrs. Part O, **223**(6), pp 199-214.
16. Carter, A. D. S. (1997) Mechanical Reliability and Design. Macmillan, London.
17. Haugen, E. B. (1980) Probabilistic Mechanical Design. Wiley-Interscience, New York.
18. Smith, D. J. (1993) Reliability, Maintainability and risk: practical methods for engineers, 4th Edition, Butterworth-Heinemann, Oxford.
19. Rice, R. C. (1997) Statistical aspects of design. In ASM Handbook No. 20, Materials Selection and Design, 10th Edition, ASM International, Ohio.
20. Modarres, M., (1993) What Every Engineer Should Know about Reliability and Risk Analysis. Marcel Dekker, New York.
21. Swift, K. G., Raines, M. & Booker, J. D. (2000) Case Studies in Probabilistic Design. Journal of Engineering Design, **11**(4), 299-316.
22. Hoole, J., et al. (2016) Safe-Life Fatigue and Sensitivity Analysis: a Pathway Towards Embracing Uncertainty?", In Proc. Royal Aeronautical Society 5th Aircraft Structural Design Conference, 4-6 October, Manchester, UK.
23. Wirsching P. H., (1995) Probabilistic Fatigue Analysis, In: Probabilistic Structural Mechanics Handbook – Theory and Industrial Applications, Ed. Sundararajan, C., Chapman & Hall.
24. Socie, D. (2018) eFatigue, www.efatigue.com_ [Accessed 21/03/2018]
25. IHS ESDU Engineering Sciences Data Unit (2006), Endurance of High-Strength Steels, Data Item 04019a.
26 Booker, J. D. et al (2004) A Comparison of Shrink-fit Holding Torque using Probabilistic, Micro-mechanical and Experimental Approaches. Proc. Instn. Mech. Engrs., Part B, **218**, 175-187.
27 Zentuti, N. A. et al. (2017) A review of probabilistic techniques: towards developing a probabilistic lifetime methodology in the creep regime. Materials at High Temperatures, **34**(5-6), 333–341.
28 Shigley, J. E. & Mischke, C. R. (1989) Mechanical Engineering Design. 5th Edition, McGraw-Hill, New York.
29 Jensen, H. A. & Sepulveda, A. E. (2000) Optimal design of uncertain systems under stochastic excitation. AIAA Journal, **38**(11), 2133–2141.